ENTRANCE EXAMINATION IN PHYSICS/ MATHEMATICS FOR APPLICANTS ENTERING MASTER'S PROGRAMS OF LANDAU PHYSTECH SCHOOL OF PHYSICS AND RESEARCH

Program structure: The program consists of two parts: Mathematics and Physics. Applicants for the "Integrated Structural Biology and Genetics" competition group have the right to take the entrance test by choosing a subject to pass.

Mathematics

The entrance test in mathematics consists of a written exam (duration – 3 hours) and an oral interview. The final grade is set based on the results of the both parts of the test.

1. Limit of a numerical sequence and its properties. The Cauchy criterion. Partial limit, limit superior and limit inferior. The Bolzano-Weierstrass theorem.
2. Limit of a function of one variable and its properties. Cauchy and Heine definitions of limit, their equivalence. The Cauchy criterion.
3. Continuity of a function at a point. Properties of a continuous function on a closed interval: Weierstrass and Bolzano-Cauchy theorems. Inverse function theorem. Uniform continuity, Cantor's theorem.
4. Derivative at a point of a function of one variable and its properties. Derivative of a composite function. Differentiability of a function at a point, differentiable functions. Differentiation of an inverse function.
5. Higher-order derivatives of a function of one variable. The Leibniz formula.
6. Rolle's theorem. The finite-increment theorems of Lagrange and Cauchy (mean-value theorems).
7. Taylor's formula with the Peano and Lagrange forms of the remainder.
8. The connection between the type of monotonicity of a differentiable function and the sign of its derivative. Sufficient conditions for the presence or absence of a local extremum in terms of the first, second, and higher-order derivatives. Convex functions. Differential conditions for convexity.
9. Differentiability of a function of several variables. Necessary conditions and sufficient conditions for differentiability.
10. The implicit function theorem.
11. Local extremum of a function of several variables. Necessary conditions and sufficient conditions of local extremum.
12. Extrema with constraint (necessary condition). The method of Lagrange's multipliers, necessary and sufficient conditions for a conditional extremum.
13. Definite integral. Darboux criterion for integrability of a function. Integrals with a variable upper limit of integration, their properties: continuity, differentiability. The Newton – Leibniz formula.
14. Improper integrals. Absolute and conditional convergence. The Cauchy criterion, comparison tests and Dirichlet's test for convergence of improper integrals.
15. Numerical series. Absolute and conditional convergence. The Cauchy's criterion, the comparison theorem, integral test, Leibniz and Dirichlet tests for convergence of numerical series.
16. Functional series. Uniform convergence. The Cauchy criterion, Weierstrass and Dirichlet tests for uniform convergence.
17. Power series. Radius of convergence, Cauchy-Hadamard formula. Taylor series. Decomposition of elementary functions into Taylor's series.
18. Curvilinear integral. Green's formula.
19. Surface integral. The Ostrogradsky-Gauss and Stokes formulas.
20. The Riemann-Lebesgue Lemma. Trigonometric Fourier series for absolutely integrable functions, the tendency of their coefficients to zero. Sufficient conditions for a Fourier series to converge at a point. Uniform convergence of Fourier series.

21. The Fourier transform of an absolutely integrable function and its properties. Fourier transform of the derivative and the derivative of the Fourier transform.
22. The Weierstrass approximation theorem. Complete systems in normed spaces.
23. Different types of representations of straight lines and planes. Angles between straight lines and planes. Distance from a point to a line and a plane. Distance between skew lines.
24. Second order curves. Ellipse, parabola, hyperbola and their properties.
25. Affine transformations and their properties. The main directions of affine transformations. The geometric meaning of the absolute value and sign of the determinant of an affine transformation matrix.
26. Orthogonal transformations of a plane and its properties. Decomposition of an affine transformation into an orthogonal transformation and two shearings.
27. Systems of linear algebraic equations. Kramer's rule. Rouché–Capelli theorem. Fredholm's Theorem. General solution to a system of linear equations.
28. Linear transformation of a finite-dimensional space, its matrix. Change of basis. Eigenvectors and eigenvalues, their properties.
29. Quadratic forms and their reduction to the canonical form.
30. Finite-dimensional Euclidean spaces. The Gram Matrix. Conjugate linear transformation of a finite-dimensional Euclidean space and its properties.
31. Self-adjoint linear transformations of a finite-dimensional Euclidean space, properties of its eigenvalues and eigenvectors.
32. Ordinary differential equations. Separation of variables. Reduction of order of differential equations. Introducing a parameter.
33. Linear differential equations and linear systems of differential equations with constant coefficients. Finding solutions.
34. Linear differential equations and linear systems of differential equations with variable coefficients. Fundamental system of solutions. Wronskian. Liouville-Ostrogradski formula. Variation of constants.
35. The simplest problem of calculus of variations. Necessary condition for a weak local extreme, Euler equation.
36. Autonomous systems of differential equations. Classification of equilibrium points of linear autonomous systems of second-order equations. Stability and asymptotic stability of the equilibrium point.
37. First integrals of an autonomous system of differential equations. Theorem on the number of independent first integrals. Linear differential equations in partial derivatives, general solution to the Cauchy problem.
38. Probability space. Independent events. Addition theorem of probability. Conditional probability. A complete system of events. The formula of total probability. Bayes formula.
39. Random variable and its distribution. Mathematical expectation and the variance of the random variable and their properties.
40. Bernoulli scheme. Chebyshev's inequality and the law of large numbers.
41. Regular functions of a complex variable. Cauchy integral formula. Ring of regular functions. Laurent series.
42. Residues. Cauchy's residue theorem. Formula for calculating residue. Jordan's lemma. Entire functions and their properties.
43. Regular branches of multivalued complex functions $\sqrt{zn}$ and Ln(z) and their application for calculating integrals.
44. Conformal mappings. Fractional-linear mapping and its properties. Zhukovsky's function and its properties.
45. Second order linear partial differential equations in two variables that are hyperbolic in a given domain. Method of characteristics for the search of general solution and the solution of Cauchy's problem.
46. Cauchy problem for the wave equation and one-dimensional heat equation. D'Alembert and Poisson formulas.
47. Mixed problem for the wave for a semi-infinite string. Initial and boundary conditions.
48. Cauchy problem for the wave equation in three-dimensional space. Kirchhoff formula.
49. Internal and external Dirichlet and Neumann problems for Laplace and Poisson's equations

in a circle and a ball.
50. Fourier method for solving a mixed problem for the wave and heat equations.
51. Fredholm integral equations of the second kind with degenerate kernels.

Literature for self-study

1. George B. Thomas, Maurice D. Weir, Joel Hass, Frank R. Giordano. Thomas's calculus.
2. Vladimir A. Zorich. Mathematical Analysis I.
3. Vladimir A. Zorich. Mathematical Analysis II.
4. Ruslan A. Sharipov. Course of analytical geometry.
5. Jim Hefferon. Linear Algebra.
6. Ruslan A. Sharipov. Course of linear algebra and multidimensional geometry.
7. Gilbert Strang. Linear algebra and its applications.
8. W. Keith Nicholson. Linear Algebra with Applications.
9. William E. Boyce, Richard C. DiPrima. Elementary Differential Equations and boundary value problems.
10. Dmitri P. Bertsekas, John N. Tsitsiklis. Introduction to Probability, 2nd Edition.
11. Joseph K. Blitzstein, Jessica Hwang. Introduction to Probability.
12. G. Cain. Complex analysis.
13. T. Gamelin. Complex analysis.
14. Yehuda Pinchover, Jacob Rubinstein. An introduction to partial differential equations.

Physics

The entrance test consists of a written part (duration - 1.5 hours) and an interview (approximately one hour after the end of the written part). The final grade for the subject is based on the results of both parts of the test.

1. Laws Newton's law. Inertial and non-inertial reference systems.
2. The principle of relativity Galileo and the principle of relativity Einstein.Conversions The Lorentz method. Invariance of the interval.
3. Laws of conservation of energy and momentum. Elastic and inelastic collisions.
4. Equation of motion of a material point in relativistic mechanics. Momentum andenergy of a material point.
5. The law of universal gravitation and its laws Kepler. Motion of bodies in the gravitational field.
6. Law of conservation of angular momentum. Equation of moments. Rotation of a rigid bodyaround a fixed axis.
7. The flow of an ideal fluid. The continuity equation. The equation Bernoulli.
8. Viscous fluid movement. The formula Poiseuille. Number Reynolds, its physicalmeaning.
9. Elastic deformations. Young's modulus and coefficient Poisson's law. Energy of elastic deformation.
10. Equation of state of an ideal gas, its explanation на based on molecularkinetic theory. Non-ideal gas equation Van der Waals.
11. Quasi -static processes. The first law of thermodynamics. The amount of heat andwork. Internal energy. Enthalpy.
12. The second law of thermodynamics. Cycle Carnot. Entropy and the law of its increase. Entropy of an ideal gas.
13. Thermodynamic potentials. Conditions of equilibrium of systems.
14. Distributions Maxwell and Boltzmann.
15. Heat capacity. The law of uniform distribution of energy in degrees of freedom. Dependence of the heat capacity of gases on temperature.
16. Phase transitions. The equation Clapeyron-Clausius. State diagrams.
17. Transfer phenomena: diffusion, thermal conductivity, and viscosity. Transport coefficients ingases.
18. Fluctuations. Brownian motion. Ratio Einstein.
19. Coulomb's law. The Gauss theorem in differential and integral forms. Circulation theorem for an electrostatic field. Potential. The equation Poisson's law.
20. Electrostatic field in a substance. Polarization vector, electric induction. Boundary conditions for vectors E and D.
21. Magnetic field of direct currents in vacuum. Basic equations of magnetostatics in vacuum. The Bio-Savard law. Power Amperes. The Lorentz force.
22. Magnetic field in a substance. Basic equations of magnetostatics in matter. Boundary conditions for vectors B and H.
23. Electromagnetic induction in moving and stationary conductors. EMF of induction. Self- and mutual induction. The reciprocity theorem.
24. System of equations Maxwell 's equations in integral and differential forms. Tок Offset current. Material equations.
25. The law of conservation of energy for an electromagnetic field. Poynting vector. Pulse of the electromagnetic field.
26. Quasi -stationary currents. Free and forced oscillations in electrical circuits. The phenomenon of resonance. Q-factor of the oscillatory circuit, its energy meaning.

27. Spectral decomposition of electrical signals. Spectra of vibrationsmodulated by amplitude and phase.
28. Electrical fluctuations. Shotgun blast and thermal noise. Sensitivity limitof electrical measuring devices.
29. Electromagnetic waves. The wave equation. The equation Helmholtz.
30. Electromagnetic waves in waveguides. Critical frequency. Volume resonators.
31. Plasma. Plasma frequency. Dielectric constant of plasma.
32. Wave interference. Temporal and spatial coherence. Uncertainty ratio.
33. Principle Huygens-Fresnel method. Fresnel zones. Fresnel and Fraunhofer diffraction.Limits of applicability of geometric optics.
34. Spectral devices (prism, diffraction grating, Fabry interferometer Stylus) and their main characteristics.
35. Diffraction limit of the resolution of optical and spectral devices. CriterionRayleigh.
36. Spatial Fourier transform in optics. Diffraction on sinusoidallattices. Abbe's theory of image formation.
37. Principles of holography. The hologram Gabor. A hologram with an inclined reference beam. Three-dimensional holograms.
38. A wave packet. Phase and group velocity. The formula Rayleigh. Classical theoryof variance. Normal and abnormal variance.
39. Polarization of light. Angle Brewster. Optical phenomena in uniaxial crystals.
40. X-ray diffraction. The Bragg-Wolf formula. Refractive index of a substance for X-rays.
41. The quantum nature of light. External photo effect. The equation Einstein. Effect Compton.
42. Spontaneous and stimulated radiation. Inverse population of levels. The principle of laser operation.
43. Blackbody radiation . The formula Planck, laws Wines and Stefan- Boltzmann.
44. Wave-particle dualism. De Broglie waves. Devisson-Germer experiments and Thomson 's theory of electron diffraction.
45. The wave function. Coordinate and momentum operators. Average values of physical quantities. Uncertainty relation for coordinate and momentum. The Schrodinger equation.
46. Bohr's postulates. Energy spectrum of hydrogen -like atoms. Characteristic radiation, Moseley's law.
47. Stern's experiments and Gerlach. Electron spin. Orbital and spin magnetic moments of an electron.
48. Identity of particles. Symmetry of the wave function with respect to the permutation of particles. Bosons and fermions. Principle Pauli. Electronic structure of atoms. Table of contentsMendeleev.
49. Fine and hyperfine structure of optical spectra. Selection rules for the absorptionand emission of photons by atoms.
50. The Zeeman effect in weak magnetic fields.
51. The Zeeman effect in strong magnetic fields.
52. Nuclear and electronic magnetic resonances.
53. The law of radioactive decay. Half и -life and lifetime.
54. Tunneling of particles through a potential barrier. Alpha decay. Geiger's Law- Nattola and his explanation.
55. Types of beta decays. Explanation of the continuity of the electron energy spectrum.Neutrinos.

56. Nuclear reactions. Composite core. Cross -section of non-resonant reactions. Bethe's law.
57. Resonant nuclear reactions, formula Breit-Wigner.
58. Nuclear fission under the action of neutrons. The principle of operation of athermal neutron nuclear reactor.
59. Uncertainty relation for energy and time. Estimation of the lifetimeof virtual particles and the radii of strong and weak interactions.
60. Fundamental interactions and fundamental particles (leptons, quarks, andcarriers of interactions). Quark structure of hadrons.

Literature for self-study

1. Sivukhin D. V. General course of physics. Vol . 1-5 , Moscow: Fizmatlit Publ., 2003.
2. Collection of problems in the general course of physics. Vol. 1-3 / под ed. V. A. Ovchinkin. – Moscow: Fizmatkniga Publ., 2013.
3. Kingsep A. S., Lokshin G. R., Olkhov O. A. Osnovy fiziki [Fundamentals of physics]. Course of General Physics, Vol. 1-2, Moscow: Fizmatlit Publ., 2001